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Discovery teaching about optimization problems in economics

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Abstract

At Ho Chi Minh University of Banking in particular and universities in the economic sector in general, there is a remarkable innovation in the way of teaching Advanced Math 2 (Analytic Mathematics) to students. Instead of focusing on imparting pure advanced math knowledge as before, teachers began to teach students maths in economics. Students are excited and the teaching process is therefore significantly improved. Among the teaching methods that promote students' activeness, discovery teaching is the teaching method we choose to impart advanced math knowledge to students. Discovery learning maximizes its advantages. It helps students to be confident, overcome their limitations and stimulate their eagerness to learn and give their opinions enthusiastically. In this article, we learn about the perspectives, characteristics, advantages, and limitations of discovery teaching. Besides, we also cover the levels as well as the discovery learning process. In addition, we also provide illustrative examples of the discovery method of teaching for optimization problems (extreme problems) in economics. Through theoretical, practical, and observational research, the research shows the effectiveness of applying discovery teaching in the second advanced math module (analytic math).

Keywords: Discovery teaching, extreme problems, Lagrange functions, economics, perspectives, processes.

INTRODUCTION

Discovery teaching is an active and modern teaching method that has many benefits in attracting interest and initiative, curiosity, learning, and practice

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in students. This teaching method will help change the way students learn math and develop higher-order thinking at work. Students are engaged, self-directed, and responsible for presenting their own work (Rooney, 2009). It comes in many shapes and sizes. The common point of discovery teaching is to make learners deeply involved in math activities. When faced with math problems, students need to know how to solve and develop mathematical ideas. Discovery activities often do not know the answer in advance. Learners often find it difficult to find solutions to problems (Ernst et al., 2017). Learners are often stimulated with curiosity by questions provided by teachers or created by learners themselves. Learners build new knowledge and understanding on a solid foundation of old knowledge. Students are the center of the teaching process; teachers only play a supporting role. Learners have a high responsibility in finding knowledge for themselves. The learning outcomes of discovery learning often include the development of skills for self-reflection, critical thinking, the ability to make an independent discovery, and being responsible for oneself for intellectual growth and maturity. (Spronken-Smith & Walker, 2010). In this article, we focus on answering the following questions:

1. What is the perspective of discovery teaching?
2. What are the levels of discovery teaching?
3. What are the advantages and disadvantages of discovery teaching?
4. What is the process of discovery teaching?
5. What are examples of discovery teaching in economic optimization problems?

CONTENTS

Perspectives on Discovery Teaching

Discovery learning is a promising teaching model. Teachers are facilitators of students' learning experiences to achieve their learning goals. Students are responsible for their own learning (Feletti, 1993). It is a method that is geared towards self-directed learning activities (Hmelo-Silver et al., 2007). Teaching methods include activities of learners to open up opportunities to explore and gain knowledge by themselves (Panasan & Nuangchalerm, 2010). Teaching has positive effects, fostering deep understanding for students, helping students to achieve higher competence, making the acquisition of new knowledge better (Friesen & Scott, 2013).

This is a pedagogical approach that plays an active and important role in enhancing higher-order cognitive skills in students. It promotes learning motivation and helps students form new knowledge and new processes. In this mode of teaching, the teacher plays an integral role in questioning surveys. Students are the center, the main operator to carry out discovery activities. Students develop critical thinking, problem-solving

abilities, and better integration of knowledge than traditional methods. Students have a better connection between applying knowledge and real-life (Madhuri et al., 2012). It can be said that the discovery teaching method will improve students' understanding. It helps students develop curiosity, creativity in math and science, as well as the potential for reflection, reasoning and critical analysis, and autonomy as learners (Harlen, 2013).

Discovery teaching is one of the most challenging and exciting teaching methods in schools today. Students participate in self-directed learning activities supported by teachers. This way of learning will boost students' motivation and interest in the subject (Bell et al., 2010). Learners participate in many activities and thought processes to create new knowledge. Teachers are encouraged to replace traditional teacher-centered teaching methods, such as emphasis on textbooks, lectures, and scientific facts, with discovery-oriented approaches to attract students' interest in teaching activities. Teachers create opportunities for students to use logical thinking, give evidence, and encourage students to explain and give answers based on their own arguments (Abdi, 2014). The teaching environment provides students with the opportunity to interact with specific areas of knowledge. Discovery-based learning engages students in meaningful activities, strengthens critical thinking skills, helps students develop the ability to investigate, identify alternative ideas, and stimulate students to self-study. It stimulates students' higher-order thinking processes, allowing students to conceptualize a problem and find relevant explanations for that problem. Students engage in interpretation, reflection, and reinforcement of thinking through meaningful activities to solve problems (Hwang et al., 2015). Students design studies, collect information, analyze data, and build evidence… Then, they debate the conclusions they have drawn from their evidence. In fact, students build and debate theories… Questions, theories, and arguments form the structure of students' scientific activities. Students conduct their own investigations, such as asking questions, planning activities, and drawing conclusions about what they have learned (Kuhn et al., 2000).

The teaching method is based on emphasizing the active participation and responsibility of learners in discovering new knowledge. In this process, students often engage in self-directed learning, partly inductive and deductive (Pedaste et al., 2015). The teaching method encourages students to participate in activities in which students explore, construct and evaluate knowledge (Wu & Hsieh, 2006). Students naturally explore and learn about the learning environment. In the classroom, students answer research questions, construct their own knowledge, and develop their understanding with teacher support. Discovery teaching promotes higher-order thinking and metacognitive skills essential for meaningful learning; enhances interest and motivation; has rich experience of problems, provides

timely information, personalizes problem situations, monitors learning processes, and improves the effectiveness of both problem situation creation and problem-solving (Wang et al., 2010).

Levels of Discovery Teaching

A four-level model illustrates how discovery teaching is introduced by reducing teacher assistance to students. The levels in this model are structured exploration question, guided discovery question, open discovery question, and combined exploration question respectively.

Table 1. The Required Level of Discovery Teaching

Levels	Description
Structured	There is direct instruction from the teacher. Students follow the teacher's instructions in their scientific investigation to create a specified number of finished products. For example, they investigate a teacher-provided discovery question through teacher-defined procedures and receive detailed step-by-step instructions for each stage of the exploratory investigation.
Guided	There is guidance from the teacher but less than a structured level. Students are responsible for establishing the direction and methods of their investigation. Teachers help students develop surveys, for example by presenting a set of possible survey questions for students to choose from and suggesting methodological guidance.
Open	Students do it themselves. Students take the lead in question formulation and investigative methods and receive teacher support. For example, students begin the investigation process by asking scientific questions and making their own decisions about the design and conduct of the investigation, and communicating the results.
Coupled	A combination of two levels, for example a guided investigation phase followed by an open investigation phase.

(Rooney, 2009)

Advantages and Disadvantages of Discovery Learning

Advantages of discovery learning

- Discovery teaching makes students active in the learning process. When participating in learning, students pay more attention.
- Discovery learning fosters curiosity.
- Discovery learning promotes the development of high academic skills in social life.
- Discovery learning allows for personalized learning experiences.
- Discovery learning is highly motivating because it allows individuals the opportunity to experience and discover something for themselves.
- Discovery learning is first built on the knowledge and understanding of students.
- Discovery teaching focuses students' attention on important ideas or important techniques.

- Discovery teaching forces students to always respond, and this feedback in information processing becomes much more profound than mere memorization.
- Discovery teaching provides students with the opportunity to receive quick feedback on their understanding.
- Discovery learning allows students to connect information with facts to create a stimulus for information retention.
- Discovery learning is motivating. It is capable of combining individuals' desire for successful problem solving with information recall (Nguyen, 2016).

Limitations of discovery teaching

- Discovery teaching has the potential to be confusing for students if they don't have an initial knowledge base.
- Discovery teaching has practical limitations as schools do not consider it as the main teaching method for students to learn lessons.
- Discovery teaching is not effective. It is too time-consuming to do the lesson activities (eg math operations). There won't be enough time for students to "discover" everything during the school year.
- Discovery teaching requires teachers to prepare a lot for error correction, a lot of feedback about students making mistakes (trial and error process).
- Discovery teaching can become a barrier, that is, there are so many important skills and information that all students should learn.
- If discovery teaching is taken as the most important educational theory, it tends to produce an incomplete education.
- Discovery teaching in the traditional classroom is only possible with a small number of students, not interacting with students in different geographical areas, such as students in this province, that province, or this country, another country. The interactive environment in traditional discovery teaching is limited.
- Traditional discovery teaching with a large number of students doesn't have enough experts to help with the immediate feedback phases. When students choose the wrong option, traditional discovery teaching does not immediately provide additional information or guidance to students.
- Traditional discovery teaching often requires a teacher to perform the teaching phase. Students explore according to the teacher's activities and requests (Nguyen, 2016).

Discovery Teaching Process

Discovery learning has many different teaching processes. Levy & Petrulis (2012) suggest four steps of discovery teaching: *identifying, pursuing, producing, and authoring.*

- *Authorizing:* Questioning tasks are designed to encourage students to explore open-ended questions, problems, situations, or their own questions, avenues of discovery, and interaction with knowledge (how can I answer my open-ended question?).
- *Producing:* Q&A tasks are designed to encourage students to explore open-ended questions, problems, situations, or their own open-ended questions, framed by the teacher or others, for example external 'customers', discovery paths, and knowledge base interactions (how to answer open-ended questions?).
- *Pursuing:* Q&A tasks are designed to encourage students to actively explore their knowledge base by pursuing their own questions, problems, situations, or paths of discovery (What is the existing answer to my question?).
- *Identifying:* Question and answer tasks are designed to encourage students to actively explore the knowledge base in response to questions, problems, situations, or modes of discovery framed by the teacher (What is the existing answer to this question?).

(Levy & Petrulis, 2012).

Bell et al. (2010) offers another discovery teaching process.

- *Orientation and questioning* are almost always the first steps of an investigative process. Students observe or look at issues that interest or arouse students' curiosity. Ideally, they develop the questions themselves. A specific difficulty in the area to be explored is formulating “good” and relevant questions. Getting good questions often takes some trying. Because after some attempts, insight in this area increases.
- *A hypothesis* is the construction of relationships between variables. Making a hypothesis is a difficult task for many students. In the early stages of learning, students often don't know what to focus on. Another problem is that students don't know what a hypothesis should be. They don't realize that it includes variables and the relationships between them.
- *Planning* involves testing the hypothesis and selecting the appropriate measurement tools to determine the validity of the hypothesis. In open inquiry, learners have the opportunity to organize their own learning sometimes independently of the teacher, which requires the use of a number of organizational, control, and supervisory strategies known as process management strategies or specified procedures.

- *Investigating* includes the use of tools to collect information and data. The types of information and data required vary widely between sectors and also depend on the investigation process.
- *Analysis and interpretation* lead to the confirmation of the current hypothesis. Another obstacle for learners seems to be the interpretation of graphs, for example, the result of a computer simulation.

On the basis of reference to Edelson et al., (1999), we propose a discovery teaching process including the following steps:

- *Step 1. Pose the problem*
  This step includes inquiry activities that can lead students to face the boundaries of their knowledge or to recognize gaps in that knowledge. Students can ask questions or ask themselves questions to determine the problem they need to solve. How can I solve the problem?
- *Step 2. Explore and adjust*
  This step includes problem-solving activities, reframing the problem so that it is easy to solve. Where has the same problem students encountered been solved? Was there any way to solve this problem before?
- *Step 3. Conclusion*
  This step includes activities to draw comments after solving the problem. Can the problem be developed? Is it possible to generalize the problem? Is there a similar problem with a similar solution? How can the problem be exploited and transformed?
- *Step 4. Deepen the problem (if any)*
  This step includes new problem-solving activities that are either self-provisioned or suggested by the teacher.

Illustrative Examples of Discovery Teaching in Economic Optimization Problems

Example 1. A manufacturer needs to rent empty land near the sea to store imported materials. They rent $100\,000\ m^2$ and the yard is rectangular in shape. The rental cost is $C = 5000x + 2000y$ in some currency.

a) Find x so that $C_{\min}$.

b) Find $C_{\min}$.

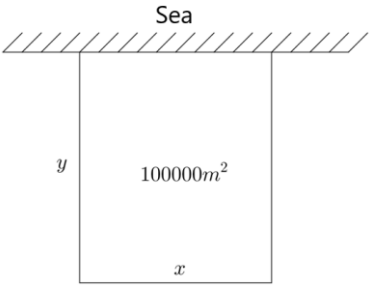

Figure 1. Minimum Cost (Source: Personal Collection)

(Tran, 1999)

Solution

- Step 1. Pose the problem

Find x so that $C_{\min}$ *and find* $C_{\min}$.

- Step 2. Explore and adjust

Since $xy = 10\,000$, we have $\varphi(x, y) = xy - 100\,000 = 0$ and the Lagrange function is $L(x, y, \lambda) = 5000x + 2000y + \lambda(xy - 100\,000)$.

The first derivative is

$$\begin{cases} L_x'(x, y, \lambda) = 5000 + \lambda y = 0 \\ L_y'(x, y, \lambda) = 2000 + \lambda x = 0 \\ xy = 100\,000 \ (x, y > 0) \end{cases} \Leftrightarrow \begin{cases} y = -\dfrac{5000}{\lambda} \\ x = -\dfrac{2000}{\lambda} \\ \dfrac{10000000}{\lambda^2} = 100\,000 \ (x, y > 0) \end{cases}$$

$$\Rightarrow (x, y, \lambda) = (500, 200, -10).$$

So the extreme point in doubt is $(500, 200, -10)$.

The second derivative is

$$L_{xx}''(x, y, \lambda) = 0,\ L_{xy}'' = \lambda,\ L_{yy}'' = 0$$

The Hessian matrix is

$$\overline{H} = \begin{bmatrix} 0 & \varphi_x' & \varphi_y' \\ \varphi_x' & L_{xx}'' & L_{xy}'' \\ \varphi_y' & L_{xy}'' & L_{yy}'' \end{bmatrix} = \begin{bmatrix} 0 & y & x \\ y & 0 & \lambda \\ x & \lambda & 0 \end{bmatrix}$$

With the extreme point in doubt of $(500, 200, -10)$, we have

$$|\overline{H}| = \begin{vmatrix} 0 & 200 & 500 \\ 200 & 0 & -10 \\ 500 & -10 & 0 \end{vmatrix} = -2\,000\,000 < 0.$$

- Step 3. Conclusion

a) $x = 200$ (Money unit), we have C_{min}.

b) $C_{\min} = 5000 \cdot 200 + 2000 \cdot 500 = 1000\,000 + 1000\,000 = 2\,000\,000$ (money unit).

- Step 4. Deepen the problem (if any)

First, we find some other solutions to the problem.

The second way

Let x and y be the width and length of the rectangular field, respectively. We have $C = 5000x + 2000y$ such that xy = 100000 is satisfied.

From here, we draw

$$C = 5000x + 2000 \cdot \frac{100\,000}{x} = 5000x + \frac{200\,000\,000}{x} \geq 2\sqrt{5000x \cdot \frac{200\,000\,000}{x}}$$
$$= 2\sqrt{1000\,000\,000\,000} = 2\,000\,000$$

The sign "=" occurs when $5000x = \frac{200\,000\,000}{x} \Rightarrow x = 200.$

a) $x = 200$ (Money unit), then we have C_{min}.

b) $C_{\min} = 2\,000\,000$ (Money unit).

The third way

Let x, y be the width and length of the rectangular field, respectively. We have $C = 5000x + 2000y$ such that xy = 100000 is satisfied.

Let a be the shortest length of the field, we have

$$C = 5000x + 2000 \cdot \frac{100\,000}{x} - a \geq 0 \Leftrightarrow 5000x^2 - ax + 200\,000\,000 \geq 0$$
$$\Leftrightarrow x^2 - \frac{a}{5000}x + 40\,000 \geq 0$$
$$\Leftrightarrow (x - \frac{a}{10\,000})^2 + 40\,000 - \frac{a^2}{100\,000\,000} \geq 0$$
$$\Leftrightarrow a^2 \leq 4000\,000\,000\,000$$
$$\Leftrightarrow a \leq 2000\,000 \Rightarrow a = 2\,000\,000,\ x = 200.$$

a) $x = 200$ (Money unit), then we have C_{min}.

b) $C_{\min} = 2\,000\,000$ (Money unit).

The fourth way

Let x, y be the width and length of the rectangular field, respectively. We have $C = 5000x + 2000y$ such that xy = 100000 is satisfied.

From here, we draw

$$C = 5000x + 2000 \cdot \frac{100\,000}{x} = 5000x + \frac{200\,000\,000}{x}.$$

The first derivative is

$$C' = 0 \Leftrightarrow 5000 - \frac{200\,000\,000}{x^2} = 0 \Rightarrow x = 200\ (x > 0).$$

The second derivative is

$$C'' = \frac{400\,000\,000}{x^3} > 0.$$

a) $x = 200$ (Money unit), then we have C_{min}.

b) $C_{\min} = 2\,000\,000$ (Money unit).

We turn the problem upside down and state example 1 as follows:

Example 2. People use x m width and y m length to fence an area of land into a rectangle close to the sea (that is, this side facing the sea is not fenced). We know that the fenced width and length are different. The cost of fencing 1 meter wide is 5000 units of money, the cost of fencing 1 meter length is 1000 money units and the total cost is $2\,000\,000$ money units. What is the largest area of land that the person can fence?

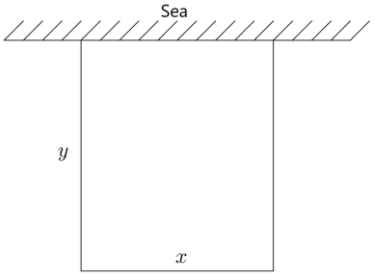

Figure 2. The Largest Land Area (Source: Personal Collection)

- Step 1. Pose the problem

Find $(xy)_{\max}$

- Step 2. Explore and adjust

Let x, y be the width and length of the rectangular field, respectively. We have $S = xy$ such that $5000x + 2000y = 2\,000\,000 \Leftrightarrow 5x + 2y = 2000$ is satisfied.

We have $\varphi(x, y) = 5x + 2y - 2000 = 0$ and the Lagrange function is

$$L(x, y, \lambda) = xy + \lambda(5x + 2y - 2000).$$

The first derivative is

$$\begin{cases} L'_x(x, y, \lambda) = y + 5\lambda = 0 \\ L'_y(x, y, \lambda) = x + 2\lambda = 0 \\ \varphi(x, y) = 5x + 2y - 2000 = 0 \end{cases} \Leftrightarrow \begin{cases} y = -5\lambda \\ x = -2\lambda \\ -10\lambda - 10\lambda - 2000 = 0 \end{cases} \Leftrightarrow \begin{cases} y = 500 \\ x = 200 \\ \lambda = -100 \end{cases}$$

So the extreme point in doubt is $(500, 200, -100)$.

The second derivative is

$$L''_{xx}(x, y, \lambda) = 0,\ L''_{xy} = 1,\ L''_{yy} = 0$$

The Hessian matrix is

$$\overline{H} = \begin{bmatrix} 0 & \varphi_x^{'} & \varphi_y^{'} \\ \varphi_x^{'} & L_{xx}^{''} & L_{xy}^{''} \\ \varphi_y^{'} & L_{xy}^{''} & L_{yy}^{''} \end{bmatrix} = \begin{bmatrix} 0 & 5 & 2 \\ 5 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

With the extreme point in doubt of $(500, 200, -100),$ we have

$$|\overline{H}| = \begin{vmatrix} 0 & 5 & 2 \\ 5 & 0 & 1 \\ 2 & 1 & 0 \end{vmatrix} = 20 > 0.$$

- Step 3. Conclusion

Vậy $(xy)_{max} = 500 \cdot 200 = 100\,000$ (m^2).

- Step 4. Deepen the problem

We find some other solutions to the problem.

The second way

Let x and y be the width and length of the rectangular field, respectively. We have $S = xy$ such that $5000x + 2000y = 2\,000\,000 \Leftrightarrow 5x + 2y = 2000$ is satisfied.

From here, we draw

$$S = x \cdot \left(\frac{2000 - 5x}{2}\right) = \frac{1}{10} \cdot 5x \cdot (2000 - 5x) \le \frac{1}{10}\left(\frac{5x + 2000 - 5x}{2}\right)^2 = 100\,000.$$

So $(xy)_{max} = 100\,000$ (m^2) when $x = 200,\ y = 500.$

The third way

Let x and y be the width and length of the rectangular field, respectively. We have $S = xy$ such that $5000x + 2000y = 2\,000\,000 \Leftrightarrow 5x + 2y = 2000$ is satisfied.

Let S_0 be the largest possible area of the field, we have

$$S = x \cdot \left(\frac{2000 - 5x}{2}\right) - S_0 = \frac{1}{2} \cdot (2000x - 5x^2 - 2S_0) \le 0$$

$$\Leftrightarrow \frac{5}{2}(400x - x^2 - \frac{2}{5}S_0) \le 0$$

$$\Leftrightarrow -\frac{5}{2}(x^2 - 400x + \frac{2}{5}S_0) \ge 0$$

$$\Leftrightarrow -\frac{5}{2}\left[(x - 200)^2 + \frac{2}{5}S_0 - 40\,000\right] \ge 0$$

$$\Leftrightarrow -\frac{5}{2}(x - 200)^2 + 100\,000 - S_0 \ge 0$$

$$\Leftrightarrow 100\,000 - S_0 \ge \frac{5}{2}(x - 200)^2 \ge 0$$

$$\Rightarrow S_0 \le 100\,000.$$

So $(xy)_{max} = 100\,000$ (m²) when $x = 200,\ y = 500.$

The fourth way

Let x and y be the width and length of the rectangular field, respectively. We have $S = xy$ such that $5000x + 2000y = 2\,000\,000 \Leftrightarrow 5x + 2y = 2000$ is satisfied.

Let S_0 be the largest possible area of the field, we have

$$S = x \cdot \left(\frac{2000 - 5x}{2} \right) = = -\frac{5}{2}(x^2 - 400x).$$

The first derivative is

$$S' = 0 \Leftrightarrow -\frac{5}{2}(2x - 400) = 0 \Leftrightarrow x = 200,\ y = 500.$$

The second derivative is

$$S'' = -5 < 0.$$

So $(xy)_{max} = 100\,000$ (m²) when $x = 200,\ y = 500.$

CONCLUSION

Discovery teaching is an active teaching method for learners. Discovery learning helps students create their own knowledge. Students seem to remember longer when they are allowed to explore, exploit and develop problems by themselves. Among the mathematical forms, the optimal mathematical form in economics proves to be quite suitable for discovery teaching. Optimization problems usually use the derivative to solve them. There are many methods to solve optimization problems in economics. We can solve problems using the Lagrange method, the inequality method, or the derivative method. Each method has its own advantages and disadvantages. However, for students majoring in economics, the Lagrange method is the number one preferred method. After completing the chapter on optimization problems, students should have no problem solving these optimization problems. Our article has answered all the questions raised. These are the perspectives on discovery teaching, the levels of discovery teaching, the advantages and limitations of discovery teaching, the process of discovery teaching, and illustrative examples of discovery teaching in economic optimization problems. Today, thanks to the strong development of computer science, optimization problems are of special interest, including the field of artificial intelligence. It can be said that the optimization problem is one of the "blood vessels" of artificial intelligence. Without solving the optimization problem, there is no such thing as the development of artificial intelligence today. These things prove one thing; the article really has scientific and practical significance as well as interdisciplinary significance.

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