

## Estimating the Effect of the Shape Parameter for Probability Distribution Using the Minimum Variance Method

Dr. HAMZA IBRAHIM HAMZA

Faculty of Urban Sciences

Al - Zaeem Al - Azhari University

### Abstract:

*The objective of this paper was to identify the value of the parameters of the shape of the binomial, bias one and natural distributions. Using the least variance method and identifying the criterion of differentiation to estimate the shape parameter between the probability distributions and reaching the best estimate of the parameter of the shape when the sample sizes are small, medium and large So that this estimate is as close as possible to the value of the shape parameter of the distribution concerned. The problem was to find the best estimate of the parameters of the society to be estimated so that they are close to the estimated average of the mean error squares. Different samples at different parameter values the descriptive and inductive method was selected in the analysis of the data by generating 1,000 random numbers of different sizes using the simulation method through the MATLAB program. A number of results were obtained, (0.7), (0.9), the binomial can be used because it is better for the double-edged than for the binomial distribution. Between distributions, at sample size (70) to estimate the small shape parameter (0.3) for And Zaat binary binomial and Poisson and natural and Qama, they can use the normal distribution because it is the best among distributions, and it is to estimate the shape parameter (0.5), (0.7), (0.9), it can be used as a binary binomial because it is the best binomial duo among distributions. The paper also issued a number of recommendations, most notably the use of binomial distribution to estimate the parameter of the figure (0.9) at the size of the sample (10), (30), (50), (70)*

**Key words:** effect of the shape parameter, probability distribution, minimum variance method

### **Introduction:**

The estimation of descriptive statistical measures in terms of one of the random samples taken is one of the main reasons for analysis and decision-making. Two types of estimates can be distinguished: the point estimate is only one value for the statistical constant estimated in terms of the corresponding statistical factor, calculated from the random sample drawn from the society. The estimate is to estimate the value of the statistical constant within a given field at a given probability in terms of the corresponding statistical function. Those hard into account the value of the standard error of the estimate continued to be the statistical Bdalalth value we get edged located between the highest and lowest of the most important reasons that help in the study of sampling theory is the desire to obtain information about the required study of society.

### **The Study Problem:**

When using any method of estimation, it is necessary to find the best estimate of the characteristics of the society to be estimated so that they are close to the estimate with the least error, since there is a set of distributions used in applied research.

1. What is the best estimate of the least error to estimate the shape parameter of the distributions?
2. Is it possible to know the best estimate of a distribution parameter by comparing several estimates using the least variance method?
3. What is the effect of the Minimum variance method on estimating the shape parameter of the distributions at different sample sizes at different parameter values?

### **The Importance of Studying:**

The importance of the study was as follows:

1. To emphasize the importance of compiling the data in the sizes of different samples and different probability distributions with different shape parameters, in order to avoid any problems that researchers may encounter when conducting their research on real data.
2. The importance of using the sample data in estimating the parameters of the shape of binomial, bias one and natural distributions. They used the least variance method to find out the effect of this method on the shape parameter in terms of good estimation.
3. Estimation of the point is the best estimate of the community parameter and it is the basis of the estimation process in the period and the tests of hypotheses.

### **Objectives of the Study:**

The objectives of the study were as follows:

1. Recognition of the values of the parameters of the shape of binomial, bias one and natural distributions, using the maximum potential method.
2. Recognition of the criterion of differentiation to estimate the shape parameter in the manner of the lowest variance between binomial, bias one and natural distributions.
3. To arrive at the best estimate of the shape parameter when it is small, medium and large for binomial, bias one and natural distributions so that this estimate is as close as possible to the value of the shape parameter of the distribution concerned.

### **Methodology Of The Study:**

The descriptive approach was followed with regard to the theoretical aspect of the subject of the study. As for the applied side, the case study was used to generate the sample data by the simulation method. The data were generated by binomial, Poisson, Norma, gamma distributions. Using the Mental program

### **Simulation style:**

In some cases, simulation is seen as the method that is often used when all other methods fail and the method of simulation is based on

finding the means by which the researcher can study the problem and analyze it despite the difficulties in expressing it in mathematical model<sup>(4)</sup>.

The simulation of the real system is carried out by a theoretically predictable system of behavior through a specific probability distribution. Thus, a sample of this system can be sampled by so-called random numbers<sup>(5)</sup>.

Simulation is defined as a numerical technique used to perform tests on a numerical computer that includes logical and mathematical relationships that interact with each other to describe the behavior and structure of a complex system in the real world and are finally described as the process of creating the spirit of reality without achieving this reality at all<sup>(4)</sup>.

### **The Concept of Monte Carlo Model:**

The basis of this model is the selection of the hypothesis elements available (probability) by taking random samples and can be summarized in the following steps<sup>(6)</sup>:

1. Determine the probability distribution for each variable in the model to be studied.
2. Use random numbers to simulate probability distribution values for each variable in the previous step.
3. Repeat the process for a set of attempts.

### **Random Numbers:**

Is the number was chosen by random quantity operation and random numbers are used to generate simulation values for many probability distributions. There are many ways to generate random numbers such as linear matching, use random number tables, and use functions ready for this purpose, such as the Rand function used in many programming languages<sup>(1)</sup>.

### **Maximum Variance<sup>(4)</sup>:**

Is one way of estimation, let  $x$  be a random variable with a probability block function  $p(x)$ . Which requires the estimation of any  $f(X)$ . These functions depend on the parameter  $\theta$  which requires estimating any  $\hat{\theta}$ . Therefore, we take a sample of the independent

random variables  $x_1, x_2, x_3, \dots, x_n$  for the estimation process, provides a method for estimating the least variation on the following formula:

$$\frac{\partial \ln L(x_1, x_2, x_3, \dots, x_n)}{\partial \theta} = A(\theta) [\hat{\theta} - \theta]$$

where:

The maximum likelihood function  $\equiv L(x_1, x_2, x_3, \dots, x_n)$ .

$\hat{\theta} \equiv$  The amount that has the least variation is  $\frac{1}{A(\theta)}$

$A(\theta) \equiv$  Quantity does not depend on the parameter  $\hat{\theta}$  Ie do not rely on sample data.

$$V(\theta) = \frac{1}{A(\theta)}$$

In this way, unbiased estimates of the least variance and sufficient and necessary condition for the existence of an unbiased estimate with the least variation such as  $x$  are the possibility of forming the first partial derivative of the function of the distribution function (function) as follows (5):

$$\frac{\partial \ln L(x_1, x_2, x_3, \dots, x_n)}{\partial \theta} = \frac{\tau - \theta}{\lambda}$$

It is then said that  $(\tau)$  is an unambiguous estimate with the least variance of parameter  $\theta$  and that  $\lambda$  represents the variance of this estimate.

### **Binomial distribution (7):**

The binomial distribution is one of the intermittent distributions of great practical importance in the randomized experiments that result in one of two outcomes: the initial name - the desired success and the other the non-required failure. This distribution was discovered in 1700 by the world (James Bernolli)

It is said that the variable  $x$  follows binomial distribution with  $(n, P)$  parameters if its probability attribute is:  $f(x, P, n) = C_x^n P^x (1 - P)^{n-x} = C_x^n P^x (q)^{n-x}$ ,  $x = 0, 1, \dots, n$

This distribution is represented by the symbol  $X \sim B(n, P)$ .

### **Estimating the shape parameter of the Binomial distribution in the minimum variance method:**

When estimating the **Binomial distribution** parameter in the greatest possible way:

$$\frac{\partial \ln L(x_1, x_2, x_3, \dots, x_n)}{\partial P} = \frac{\sum_{i=1}^n x_i}{P} - \frac{n^2 - \sum_{i=1}^n x_i}{1 - P}$$

And the unification of denominations :

$$\frac{\partial \ln L}{\partial P} = \frac{(1 - P) \sum_{i=1}^n x_i - n^2 P + P \sum_{i=1}^n x_i}{P(1 - P)}$$

$$\therefore \frac{\partial \ln L}{\partial P} = \frac{\sum_{i=1}^n x_i - P \sum_{i=1}^n x_i - n^2 P + P \sum_{i=1}^n x_i}{P(1 - P)}$$

$$\therefore \frac{\partial \ln L}{\partial P} = \frac{\sum_{i=1}^n x_i - P \sum_{i=1}^n x_i - n^2 P + P \sum_{i=1}^n x_i}{P(1 - P)}$$

$$\frac{\partial \ln L}{\partial P} = \frac{\sum_{i=1}^n x_i - n^2 P}{P(1 - P)}$$

But  $\bar{x} = \sum_{i=1}^n x_i$

$n^2$ : By divided on

$$\frac{\partial \ln L}{\partial P} = \frac{\frac{\bar{x}}{n} - P}{\frac{P(1 - P)}{n}}$$

And compared to the equation  $\frac{\partial \ln L(x_1, x_2, x_3, \dots, x_n)}{\partial \theta} = \frac{\tau - \theta}{\lambda}$  is being:

$$\hat{P} = \frac{\bar{x}}{n}$$

### Characteristics of Distribution:

1. Mean  $E(X) = P$
2. Variance  $V(X) = npq$

### Poisson distribution<sup>(1)</sup>:

Is a probability distribution that is used to calculate the probability of a certain number of successes (x) in a unit of time or in a given area when events or successes are independent of one another and when the average of success remains constant for the unit of time. By the world (Poisson).<sup>(1)</sup>

If (x) random variable follows the Poisson distribution with parameter  $\lambda$ , its probability function is:

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, \dots, \infty$$

Where :

$X \equiv$  number of successes.

$P (X = x) \equiv$  The probability of the given number of successes taking the value x

$\lambda \equiv$  Average number of successes per unit of time

$e \equiv$  The basis of the natural logarithmic system  $e = 2.71828$   
 This distribution is represented by the symbol  $X \sim \text{Pos}(\lambda)$ .

**Estimating the shape parameter of the Poisson distribution in the least contrast method:**

When estimating the Poisson distribution parameter in the greatest possible way:

$$\frac{\partial \ln L}{\partial \lambda} = -n + \frac{\sum_{i=1}^n x_i}{\lambda}$$

The unification of denominators:

$$\frac{\partial \ln L}{\partial \lambda} = \frac{\sum_{i=1}^n x_i - n\lambda}{\lambda}$$

By dividing  $n^2$ :

$$\frac{\partial \ln L}{\partial \lambda} = \frac{\frac{\sum_{i=1}^n x_i}{n} - \lambda}{\frac{\lambda}{n}}$$

And compared to the equation:  $\frac{\partial \ln L(x_1, x_2, x_3, \dots, x_n)}{\partial \theta} = \frac{\tau - \theta}{\lambda}$  is being:

$$\therefore \hat{\lambda} = \bar{x}$$

**Characteristics of Distribution:**

- 1. Mean  $E(X) = \lambda$
- 2. Variance  $V(X) = \lambda$

**Normal Distribution (2):**

Is one of the most frequent and most widely used probability distributions. It plays a major role in statistical theory and probability theory. This distribution was called normal distribution (or moderate or normal) because it was previously thought that any data on life should be represented and subject to this distribution, but it is now proven that this is not the case and that the belief is wrong. It is also known as the Gauss distribution, thanks to the German scientist Frederick Gauss, who developed mathematical distribution as a probability distribution in the year (1855-1777)

It is a continuous probability function, which is a gypsy shape, symmetrical around the arithmetic mean and moderate. Whenever we move away from the arithmetic mean in both directions, the normal

distribution curve approaches the horizontal axis but never touches it.<sup>(2)</sup>

If  $x$  is a random variable connected to a natural distribution of the parameters  $(\mu, \sigma)$ , the distribution function is:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty \leq x \leq \infty$$

Where:

$f(X) \equiv$  natural curved height.

$\mu \equiv$  The mean of the distribution

$\sigma \equiv$  Standard deviation

$e \equiv$  The basis of the natural logarithmic system  $e = 2.71828$ .

$\pi \equiv$  Constant ( $\pi=3.14159$ ).

This distribution is denoted by  $X \sim N(\mu, \sigma)$

### **Estimating the parameters of the shape and measurement of natural distribution in the greatest possible way <sup>(7)</sup>:**

When estimating the normal distribution parameter  $(\mu)$  in the greatest possible way

$$\begin{aligned} \frac{\partial \ln L}{\partial \mu} &= \frac{\sum_{i=1}^n (x_i - \mu)}{\sigma^2} \\ \therefore \frac{\partial \ln L}{\partial \mu} &= \frac{\sum_{i=1}^n x_i - n\mu}{\sigma^2} \end{aligned}$$

By dividing  $n$  and by the equation:  $\frac{\partial \ln L(x_1, x_2, x_3, \dots, x_n)}{\partial \theta} = \frac{\tau - \theta}{\lambda}$  is being:

$$\begin{aligned} \therefore \frac{\partial \ln L}{\partial \mu} &= \frac{\frac{\sum_{i=1}^n x_i}{n} - \mu}{\frac{\sigma^2}{n}} \\ \therefore \mu &= \frac{\sum_{i=1}^n x_i}{n} = \bar{x} \end{aligned}$$

When estimating the normal distribution parameter  $(\sigma^2)$  in the greatest possible way, we find that:

$$\frac{\partial L}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \frac{1}{2\sigma^4} \sum_{i=1}^n (x_i - \mu)^2 = 0$$

$$\begin{aligned} \therefore \frac{\partial L}{\partial \sigma^2} &= \frac{\sum_{i=1}^n x_i - n\mu}{\sigma^2} \\ &\therefore \frac{\partial L}{\partial \sigma^2} = \frac{\sum_{i=1}^n (x_i - \mu)^2}{\sigma^4} - \frac{n}{\sigma^2} \end{aligned}$$

And the unification of denominators:

$$\frac{\partial L}{\partial \sigma^2} = \frac{\sum_{i=1}^n (x_i - \mu)^2 - n\sigma^2}{\sigma^4}$$

By dividing n:

$$\frac{\partial L}{\partial \sigma^2} = \frac{\frac{\sum_{i=1}^n (x_i - \mu)^2}{n} - \sigma^2}{\frac{\sigma^4}{n}}$$

And compared to the equation:  $\frac{\partial \ln L(x_1, x_2, x_3, \dots, x_n)}{\partial \theta} = \frac{\tau - \theta}{\lambda}$  is being:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2 = s$$

### Characteristics of Distribution:

1. Mean  $E(X) = \mu$
2. Variance  $V(X) = \sigma^2$

### Gamma Distribution <sup>(2)</sup>:

It is used to study the time between the arrival of words to a particular service center, such as the arrival of the baker to a bank or the entry of patients to the hospital. A function known as  $\Gamma(\alpha)$  As follows <sup>(3)</sup>:

1. If n is positive, the integration is approximated and is:

- i.  $\Gamma(1) = 1$
- ii.  $\Gamma n = (n-1) \Gamma(n-1)$

2. If n is a positive integer, then:

$$\Gamma \alpha = (\alpha - 1) \Gamma(\alpha - 2) \times \dots \times 3 \times 2 \times 1$$

$$\therefore \Gamma \alpha = \alpha! \quad \text{or} \quad \Gamma(\alpha) \Gamma \alpha = \int_0^{\infty} e^{-X} X^{\alpha-1} dx$$

The general picture of the distribution is:

$$\Gamma\alpha = (\alpha - 1)\Gamma(\alpha - 2) \times \dots \times 3 \times 2 \times 1$$

$$\therefore \Gamma\alpha = \alpha! \quad \text{or} \quad \Gamma(\alpha)\Gamma\alpha = \int_0^{\infty} e^{-X}X^{\alpha-1} dx$$

The general picture of the distribution is:

$$f(X) = \frac{1}{\Gamma\alpha} \beta^\alpha x_1^{\alpha-1} e^{-\frac{x}{\beta}} \quad , \quad x > 0 \quad , \quad \alpha, \beta > 0$$

### Estimation of two distribution parameters in the least variance method (7):

To obtain an estimate of the measurement parameter ( $\alpha$ ) and the parameter of the form ( $\beta$ ) for the distribution in the least variance method follow the following steps:

When estimating the distribution of teachers in the greatest possible way we find that:

$$\frac{\partial L}{\partial \beta} = \frac{n\bar{x}}{\beta^2} - \frac{n\hat{\alpha}}{\beta}$$

And the unification of denominators and By dividing on ( $n\hat{\alpha}$ )

$$\frac{\partial L}{\partial \beta} = \frac{\frac{\bar{x}}{\hat{\alpha}} - \hat{\beta}}{\frac{\beta^2}{n\hat{\alpha}}}$$

And compared to the equation  $\frac{\partial \ln L(x_1, x_2, x_3, \dots, x_n)}{\partial \theta} = \frac{\tau - \theta}{\lambda}$  is being:

$$\therefore \hat{\alpha} = \frac{\bar{x}}{\hat{\beta}}$$

$$\therefore \hat{\beta} = \frac{\bar{x}}{\hat{\hat{\alpha}}}$$

### Characteristics of Distribution:

1. Mean  $E(X) = \alpha\beta$

2. Variance  $V(X) = \alpha\beta^2$

**Materials and Methods of Research:**

Distributed tracking data was generated using the Minitab program as follows:

1. Generation of the size of society M from binomial distribution  $X \sim B(n, P)$  with knowledge of n, P and distribution of Poisson  $X \sim Pos(\lambda)$  by  $\lambda$  and normal distribution  $X \sim N(\mu, \sigma)$  And the distribution of  $X \sim Gamma(\alpha, \beta)$  by  $(\alpha, \beta)$ .
2. Choose the sample size n and denote it with the symbol j.
3. Determination of parameters for both binomial distribution, Poisson, natural, and gamma.
4. Repeat steps 1-3 of  $j = 1, 2, 3$

**Monte Carlo simulation results:**

Distributed tracking data has been generated as follows:

Generate data for binomial distribution with sample sizes 10, 30, 50 and 70 and with parameter P are 0.3, 0.5, 0.7 and 0.9 and n parameter, B10.

And generate Poisson distribution trace data with sample sizes 10, 30, 50 and 70 and with parameter,  $\lambda$  are 0.3, 0.5, 0.7 and 0.9. And generate natural distribution tracking data with sample sizes 10, 30, 50 and 70 and with the parameter,  $\sigma$  0.3, 0.5, 0.7 and 0.9 and the parameter  $\mu$  (10).

And generate distribution trace data with sample sizes 10, 30, 50 and 70, with a parameter of  $\alpha$  is 0.3, 0.5, 0.7, 0.9 and  $\beta$  parameter (10).

**ANALYSIS, INTERPRETATION, AND DISCUSSION:**

**Table (1): Determination of the shape parameter for probability distribution with parameter (0.3), (0.5), (0.7) and (0.9) and sample size (10)**

Sample size	shape parameter	Estimator MSE	distribution				Less MSE
			Binomial P	Poisson $\lambda$	Normal $\sigma$	Gamma $\alpha$	
10	0.3	shape	0.19	0.5	0.225	0.446	Poisson
		MSE	0.00121	0.00004	0.00056	0.002132	
	0.5	shape	0.52	0.9	0.415	0.264	binomial
		MSE	0.000004	0.016	0.00072	0.0056	
	0.7	shape	0.77	0.1	0.61	0.786	binomial
		MSE	0.00049	0.036	0.00081	0.00074	
	0.9	shape	0.91	0.6	0.778	0.446	binomial
		MSE	0.000001	0.009	0.0015	0.0206	

Source: The Researcher by Minitab

From Table (1) we find that at the sample size 10 and the parameter of Fig. 0.3, the estimation of the shape parameter using the float method is best when parsing the Poisson because the estimated value of 0.5 is the appropriate and close to the value of the parameter of figure 0.3 and the lowest average error box MSE is equal to 0.00004 , We find that at the size of sample 10 and the parameter of Fig. 0.5, 0.7 and 0.9, the estimation of the shape parameter using the zoom method is preferable for binomial distribution because the estimated values are close to the parameter values of the shape and the mean error box MSE is equal to 0.000004 and 0.00049 and 0.000001 respectively.

**Table (2): Determination of the shape parameter for probability distribution with parameter (0.3), (0.5), (0.7), (0.9) and sample size (30)**

Sample Size	shape parameter	Estimator MSE	distribution				Less MSE
			Binomial P	Poisson $\lambda$	Normal $\sigma$	Gamma $\alpha$	
30	0.3	shape	0.3067	0.3	0.3731	0.286	Poisson
		MSE	0.00000015	0	0.00018	0.0000006	
	0.5	shape	0.48	0.533	0.4817	0.4454	Normal
		MSE	0.0000013	0.0000036	0.0000011	0.0000099	
	0.7	shape	0.7067	0.467	0.682	0.708	binomial
		MSE	0.00000015	0.0018	0.0000011	0.0000002	
	0.9	shape	0.9133	0.767	0.765	0.818	binomial
		MSE	0.00000059	0.00059	0.00061	0.00022	

Source: The Researcher By Minitab

From Table (2) we find that at the sample size 30 and the parameter of Fig. 0.3, the estimation of the parameter of the shape using the float method is best when parsing the Poisson because the value of the estimated 0.3 is equal to the value of the parameter of figure 0.3 and the lowest mean of the MSE error box is 0, We find that at the sample size 30 and the parameter of Fig. 0.5, the estimation of the shape parameter using the zoom method is better at normal distribution because the estimated value of 0.4817 is close to the value of the parameter of figure 0.5 and the lowest mean of the MSE error box equals 0.0000011. When the sample size is 30 and the parameter of Fig. 0.7 and 0.9, the estimation of the shape parameter using the float method is best when the binomial is distributed because the estimated values are close to the parameter values of the shape and the mean error box MSE is equal to 0.00000015 and 0.00000059, respectively.

**Table (3): Determination of the shape parameter for probability distribution with the parameter of (0.3), (0.5), (0.7) and (0.9) and sample size (50)**

Sample size	shape parameter	Estimator MSE	distribution				Less MSE
			Binomial P	Poisson $\lambda$	Normal $\sigma$	Gamma $\alpha$	
50	0.3	shape	0.296	0.24	0.1985	0.3107	Normal
		MSE	0.00000032	0.0000072	0.00015	0.0000002	
	0.5	shape	0.49	0.66	0.3357	0.581	binomial
		MSE	0.0000006	0.000512	0.00039	0.00013	
	0.7	shape	0.658	0.52	0.731	0.705	Gamma
		MSE	0.0000035	0.00065	0.0000014	0.00000005	
	0.9	shape	0.926	1.000	0.942	0.857	binomial
		MSE	0.0000014	0.0002	0.0000025	0.0000037	

Source: The Researcher by Minitab

From Table (3) we find that at the sample size 50 and the parameter of Fig. 0.3, the estimation of the shape parameter using the zoom method is better at normal distribution because the estimated value of 0.304 is close to the value of the parameter of figure 0.3 and the lowest average error box MSE is equal to 0.0000000, That at the sample size 50 and the parameter of figure 0.5, the estimation of the shape parameter using the zoom method is best when the binomial distribution is because the estimated value of 0.49 is close to the value of the parameter of the shape and the lowest mean of the MSE error box is equal to 0.0000006. At the sample size of 50 and the parameter of Fig. 0.7, the estimation of the shape parameter using the float method is best when dividing the object because the estimated value of 0.705 is close to the value of the shape parameter and the lowest average error box MSE is equal to 0.00000005. When the sample size is 50 and the parameter of Fig. 0.9, the estimation of the shape parameter using the zoom method is best for binomial distribution because the estimated value of 0.926 is close to the value of the parameter of the shape and the lowest mean of the MSE error box is equal to 0.0000014.

**Table (4): Determination of the shape parameter for probability distribution with the parameter of (0.3), (0.5), (0.7) and (0.9) and sample size (70)**

Sample size	shape parameter	Estimator MSE	distribution				Less MSE
			Binomial P	Poisson $\lambda$	Normal $\sigma$	Gamma $\alpha$	
70	0.3	shape	0.3114	0.3429	0.29	0.1985	Normal
		MSE	0.00000019	0.0000026	0.00000014	0.00015	
	0.5	shape	0.4857	0.4286	0.403	0.3357	binomial
		MSE	0.00000029	0.0000073	0.00013	0.00039	
	0.7	shape	0.7114	0.7143	0.5802	0.731	binomial
		MSE	0.00000019	0.00000029	0.00021	0.0000014	
	0.9	shape	0.91143	0.857	0.921	0.942	binomial
		MSE	0.00000019	0.0000026	0.00000063	0.0000025	

Source: The Researcher by Minitab

From Table (4) we find that at the sample size 70 and the parameter of Fig. 0.3, the estimation of the shape parameter using the zoom method is better at the normal distribution because the estimated value of 0.29 is close to the value of the parameter of figure 0.3 and the lowest average error box MSE is equal to 0.00000014, That at the sample size 50, 0.7 and 0.9, the estimation of the shape parameter using the zoom method is preferable for binomial distribution because the estimated values are close to the parameter values of the shape and the mean error box MSE is equal to 0.00000029, 0.00000019 and 0.00000019, respectively.

We deduce from the above that for estimating the small shape parameter (0.3) at the size of sample 70 for binomial, bias one and natural distributions, they can use the estimation method for normal distribution because it is better for normal distribution than distributions. The estimation of the shape parameter using the zoom method is best for binomial distribution because the estimated values are close to the parameter values of the binomial distribution

**RESULTS:**

1. For the size of the sample (10) to estimate the small shape parameter (0.3) for binomial and Poisson distributions, and natural, they can use the Poisson distribution because it is the best among the distributions. To estimate the parameter of the figure (0.5), (0.7) ), The binomial can be used because it is preferable to double-edged distributions.
2. In the sample size (30) to estimate the small shape parameter (0.3) for binomial distributions and Poisson and

natural, they can use the Poisson distribution because it is the best among the distributions. Among the distributions, and to estimate the shape parameter (0.7), (0.9), the binomial distribution can be used because it is the best among the distributions.

3. In the sample size (50) for estimating the small shape parameter (0.3) for the binomial distribution and Poisson and natural, they can use the normal distribution because it is the best among the distributions. Among the distributions, and to estimate the parameter of the figure (0.5), (0.9), the binomial distribution can be used because it is the best among the distributions.
4. In the sample size (70) for the estimation of the small shape parameter (0.3) for the binomial distributions and Poisson and natural, they can be used for normal distribution because it is the best among the distributions. To estimate the parameter of the figure (0.5), (0.7) ), The binomial can be used because it is preferable to double-edged distributions.

### **RECOMMENDATIONS:**

1. Use a binomial distribution to estimate the shape parameter (0.9) at sample size (10), (30), (50), (70).
2. Expand the study of a number of other community distributions.
3. Application of a number of estimation methods for binomial distributions, Poisson, natural.

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