Heating and Cooling Degree Days Approach for Determination Optimum Thermal Insulation Thickness for Building

ELTAHIR KHALID MOHAMMED ABDELGADIR  
Department of Mechanical Engineering  
University of Nyala, Sudan  

ALI MOHAMMED HAMDAN ADAM  
Department of Mechanical Engineering, University of Bahri, Sudan  

OBAI YOUNIS  
Department of Mechanical Engineering  
College of Engineering at Wadi Addwaser  
Prince Sattam Bin Abdulaziz University, KSA  

Department of Mechanical Engineering, Faculty of Engineering  
University of Khartoum, Sudan

Abstract:  
The aims of this paper to study and descript of heating and cooling degree days approach (HDD/CDD) to determine the optimum thermal insulation thickness for external wall for building under different climate regions. Because it is the common and effective technique used in analysis to determine the optimum thermal insulation thickness in different climates regions depend on different insulation materials and building structures. The degree-time method is the most common method applied to calculate heating and cooling load, with the Life Cycle Cost (LCC) analysis which is the most commonly economical analysis methods used to optimize the thermal insulation thickness of external walls. The method depends on several parameters such as the base temperature $T_b$ and, the mean daily temperature $T_o$. There is an inverse relationship between the HDD / CDD and the value of the base temperature $T_b$. So the $T_b$ increases the corresponding value of CDD / HDD decreases.
Eltahir Khalid Mohammed Abdelgadir, Ali Mohammed Hamdan Adam, Obai Younis
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Key words: optimum thermal insulation thickness, heating / cooling, degree day, life cycle cost

1. INTRODUCTION

The energy consumption is divided in four main sectors: industrial, building, transportation and agriculture. The largest energy consumer from these sectors is the building sector. Thermal insulation of building envelope is considered as an effective factor to reduce energy demand in buildings.

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{A,H}$</td>
<td>annual heating energy cost ($/m^2$)</td>
</tr>
<tr>
<td>$C_{A,C}$</td>
<td>annual cooling energy cost ($/m^2$)</td>
</tr>
<tr>
<td>$C_i$</td>
<td>cost of insulation in ($/m^3$)</td>
</tr>
<tr>
<td>$C_{i_{ext}}$</td>
<td>cost of insulation in ($/m^2$)</td>
</tr>
<tr>
<td>$C_e$</td>
<td>cost of electricity in ($/kW h$)</td>
</tr>
<tr>
<td>$C_f$</td>
<td>cost of the fuel ($/kg$)</td>
</tr>
<tr>
<td>$C_H$</td>
<td>annual heating total cost per unit area for non-insulated walls, ($/m^2$)</td>
</tr>
<tr>
<td>$C_C$</td>
<td>annual cooling total cost per unit area for non-insulated walls, ($/m^2$)</td>
</tr>
<tr>
<td>$C_{i_{ext}}$</td>
<td>annual heating total cost per unit area for insulated walls, ($/m^2$)</td>
</tr>
<tr>
<td>$C_{C_{ext}}$</td>
<td>annual cooling total cost per unit area for insulated walls, ($/m^2$)</td>
</tr>
<tr>
<td>$C_{H_{total}}$</td>
<td>total heating cost</td>
</tr>
<tr>
<td>$C_{C_{total}}$</td>
<td>total cooling cost</td>
</tr>
<tr>
<td>COP</td>
<td>cooling system coefficient of performance</td>
</tr>
<tr>
<td>$E_{A,H}$</td>
<td>annual heating load ($/m^2$-year)</td>
</tr>
<tr>
<td>$E_{A,C}$</td>
<td>annual cooling load ($/m^2$-year)</td>
</tr>
<tr>
<td>$E_{A,H_{ext}}$</td>
<td>annual heating energy cost saving per unit area, ($/m^2$)</td>
</tr>
<tr>
<td>$E_{A,C_{ext}}$</td>
<td>annual cooling energy cost saving per unit area, ($/m^2$)</td>
</tr>
<tr>
<td>CDD</td>
<td>cooling degree days</td>
</tr>
<tr>
<td>HDD</td>
<td>heating degree days</td>
</tr>
<tr>
<td>$H_u$</td>
<td>heating value of the fuel ($/kg$)</td>
</tr>
<tr>
<td>LCCA</td>
<td>Life Cycle Cost Analysis</td>
</tr>
<tr>
<td>$PWF$</td>
<td>Present worth factor</td>
</tr>
<tr>
<td>$N$</td>
<td>lifetime (years)</td>
</tr>
<tr>
<td>$PP_H$</td>
<td>payback period for heating (years)</td>
</tr>
<tr>
<td>$PP_C$</td>
<td>payback period for cooling (years)</td>
</tr>
<tr>
<td>$i$</td>
<td>interest rate</td>
</tr>
<tr>
<td>$g$</td>
<td>inflation rate</td>
</tr>
<tr>
<td>$T_0$</td>
<td>base temperature (°C)</td>
</tr>
<tr>
<td>$K$</td>
<td>thermal conductivity of the insulation material (W/mK)</td>
</tr>
<tr>
<td>$T_m$</td>
<td>mean daily temperature</td>
</tr>
<tr>
<td>$U$</td>
<td>overall heat transfer coefficient (W/m2 K)</td>
</tr>
<tr>
<td>$R_i$</td>
<td>inside air-film thermal resistances (m^2 K/W)</td>
</tr>
<tr>
<td>$R_{ins}$</td>
<td>thermal resistance of the insulation layer (m^2 K/W)</td>
</tr>
<tr>
<td>$R_{i_{ext}}$</td>
<td>outside air-film thermal resistances (m2 K/W)</td>
</tr>
<tr>
<td>$R_m$</td>
<td>total thermal resistance of the wall materials without the insulation (m2 K/W)</td>
</tr>
<tr>
<td>$R_w$</td>
<td>total thermal resistance of the wall materials without the insulation (m2 K/W)</td>
</tr>
<tr>
<td>$R_{i_{ext}}$</td>
<td>sum of $R_i$, $R_w$, $R_o$ (m2 K/W)</td>
</tr>
<tr>
<td>$x$</td>
<td>thickness of the insulation material (m)</td>
</tr>
<tr>
<td>$X_{opt,H}$</td>
<td>optimum insulation thickness minimizing the total heating cost</td>
</tr>
<tr>
<td>$X_{opt,C}$</td>
<td>optimum insulation thickness minimizing the total cooling cost</td>
</tr>
<tr>
<td>$X_{opt,H,C}$</td>
<td>optimum insulation thickness minimizing the total heating and cooling cost</td>
</tr>
<tr>
<td>$q_{A,H}$</td>
<td>annual heat losses in unit area ($/m^2$-year)</td>
</tr>
<tr>
<td>$q_{A,C}$</td>
<td>annual heat gains in unit area ($/m^2$-year)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency of the heating system</td>
</tr>
</tbody>
</table>

Thermal insulations are materials or combinations of materials that are used primarily to provide resistance to heat flow. A thermal insulator is a poor conductor of heat and has a low thermal conductivity. Insulation is used in buildings and in
manufacturing processes to prevent heat loss or heat gain. Although its primary purpose is an economic one, it also provides more accurate control of process temperatures and protection of personnel[1, 2].

There are many previous studies that used heating or cooling degree days in their studies to determine heating or cooling loads. Aynur and Figen B.[3] calculated the optimum insulation thickness of the external wall for four cities from four climate zones of Turkey, energy savings over a lifetime of 10 years and payback periods for the five different energy types and four different insulation materials applied externally on walls. Omer K. and Faruk K. [4] calculated the optimum thermal insulation thicknesses on external walls of buildings based on both annual and seasonal energy loads. This study performed for four degree-day regions of Turkey, namely, Iskenderun, Istanbul, Ankara and Ardahan. Modeste K. N. et al.[5] calculated the optimum insulation thickness, energy saving and payback period for buildings located in two climatic regions in Cameroon. Fohagui F. and, T. Ghislain [6] applied the degree-days method on determined the optimum insulation thicknesses of external wall, energy savings and payback periods with respect to the wall and insulation types. Okan Kon [7] determined the optimum insulation thicknesses using economical analyses for exterior walls of buildings. Life cycle cost (LCC) analysis and degree-day method were the approaches used for optimum insulation thickness calculations. Subhash M. et al.[8] discussed the energy saving in different type of building walls by using optimum insulation thickness with suitable insulation material. The optimum insulation thickness and energy savings are calculated by using Life-cycle cost analysis over life time of 10 years of the building.
2. HEATING OR COOLING DEGREE-DAY METHOD

One of the most common methods used in studies to calculate heating and cooling load which is the one of an input parameter used in the determination of optimum insulation thickness is the degree-time method, because annual heating /cooling energy cost is derived from annual heating or cooling loads. The degree day method is useful in diagnosing the potential impact of regional climate modifications on energy demand for space heating and cooling. The method assumes that energy needs for a building are proportional to the difference between the average outdoor temperature and a base temperature. The base temperature is the outdoor temperature above or below cooling or heating demanded [9].

Heat loss from buildings occurs through surface of external wall, window, ceiling and air infiltration [10, 11].

The heat loss per unit surface area of external wall is calculated from the equation (1)

\[ q = U \times (T_b - T_o) \]  

Where \( U \) is the overall heat transfer coefficient, \( T_b \) is the base temperature and, \( T_o \) is mean daily temperature.

The overall heat transfer coefficient \( U \) for a typical wall with insulation is given by

\[ U = \frac{1}{R_i + R_w + R_{ins} + R_o} \]  

Where \( R_i \) is the inside air film thermal resistances, \( R_w \) is total thermal resistance of the wall without the insulation, \( R_{ins} \) is the thermal resistance of the insulation layer and \( R_o \) is the outside air film thermal resistances.

\( R_o \) is determined as flowing.

\[ R_o = \frac{1}{\frac{5.8 + 4.1 \times V}{V}} \]  

Where \( V \) is air velocity in m/s.
The thermal resistance of the insulation layer \( R_{ins} \) is given by
\[ R_{\text{ins}} = \frac{x}{k} \]  
(4)

Where \( x \) is the thickness of the insulation layer and \( k \) is the thermal conductivity of the insulation material.

The annual heat losses and gains occurring in unit surface are calculated by using \( U \) and the degree-day value, \( DD \) as given by the following equations

\[ q_{A,H} = 86400 \times HDD \times U \]  
(5)

\[ q_{A,C} = 86400 \times CDD \times U \]  
(6)

Where HDD is heating degree days and CDD is the cooling degree days.

HDD is the sum of temperatures below the temperature of a certain basis temperature or another formula is the absolute value (positive) of the difference between the base temperature \( T_b \) and the average daily temperature \( T_o \), and calculated mathematically as follows:

\[ \text{HDD} = \sum_{1}^{365} |T_b - T_o| \]  
(7)

HDD is the sum of temperatures below the temperature of a certain basis temperature or another formula is the absolute value (positive) of the difference between the average daily temperature \( T_o \) and the base temperature \( T_b \), and calculated mathematically as follows:

\[ \text{CDD} = \sum_{1}^{365} |T_o - T_b| \]  
(8)

The annual energy requirement can be calculated by dividing the annual heat loss to the efficiency of the heating system \( \eta \):

\[ E_{A,H} = \frac{86400 \times HDD \times U}{\eta} \]  
(9)

Similarly, the annual cooling load can be determined in an analogous expression as

\[ E_{A,C} = \frac{86400 \times CDD \times U}{\text{COP}} \]  
(10)

If \( R_{\text{wt}} \) is the sum of \( R_i \), \( R_w \) and \( R_o \), then. If \( R_{\text{wt}} \) is the total wall thermal resistance excluding the insulation layer resistance, Eq. (2) can be rewritten as
The annual energy need for heating $E_{A,H}$ is calculated as follows:

$$E_{A,H} = \frac{86400 \times HDD}{(R_{wt} + \frac{\chi}{R}) \times \eta}$$  \hspace{1cm} (12)

The annual energy need for cooling $E_{A,C}$ is calculated as follows:

$$E_{A,C} = \frac{86400 \times CDD}{(R_{wt} + \frac{\chi}{R}) \times COP}$$  \hspace{1cm} (13)

where COP is performance coefficient of cooling system.

3. LIFE-CYCLE COST ANALYSIS LCCA AND OPTIMIZATION OF INSULATION THICKNESS

LCCA should be employed when determining the optimum insulating thickness. The life-cycle cost analysis LCCA involves the analysis of the costs of a system or a component over its entire lifetime. Total heating/cooling cost is evaluated together with the present-worth factor (PWF) for the lifetime of $N$ years. The PWF depends on the inflation rate $g$, and the interest rate $i$. According to the interest and inflation rates, PWF is defined as below.

If $i > g$:

$$r = \frac{i - g}{1 + g}$$  \hspace{1cm} (14)

If $i < g$, then

$$r = \frac{g - i}{1 + i}$$  \hspace{1cm} (15)

$$PWF = \frac{(1 + r)^N - 1}{r \times (1 + r)^N}$$  \hspace{1cm} (16)

where $N$ is the lifetime.

If $i = g$ then

$$PWF = \frac{N}{1 + i}$$  \hspace{1cm} (17)
4. ANNUAL ENERGY COST AND DETERMINATION OF THE OPTIMUM INSULATION THICKNESS

The annual energy cost for unit surface $C_{A,H}$ and $C_{A,C}$ are calculated with the following equations:

$$C_{A,H} = \frac{86400 \times HDD \times C_f}{\left( R_{wt} + \frac{x}{k} \right) \times H_u \times \eta}$$  \hspace{1cm} (18)

$$C_{A,C} = \frac{86400 \times CDD \times C_e}{\left( R_{wt} + \frac{x}{k} \right) \times COP}$$  \hspace{1cm} (19)

Where

$C_f$ in $$/kg$ is the unit fuel price.

$H_u$ is the heating value of the fuel.

$\eta$ is the efficiency of the heating system.

$C_e$ is the cost of electricity.

Cost of insulation, $C_{ins}$, in $$/m^2$, is given by

$$C_{ins} = C_i \times x$$  \hspace{1cm} (20)

where $C_i$ in $$/m^3$ is the cost of insulation material and $x$ is the insulation layer thickness.

The total heating cost of the insulated building is given by

$$C_{t,H} = C_{A,H} \times PWF + C_i \times x$$  \hspace{1cm} (21)

The total cooling cost of the insulated building is given by

$$C_{t,C} = C_{A,C} \times PWF + C_i \times x$$  \hspace{1cm} (22)

The total heating and cooling cost of the insulated building is calculated as follows:

$$C_{t,H,C} = C_{A,H} \times PWF + C_{A,C} \times PWF + C_i \times x$$  \hspace{1cm} (23)

The optimum insulation thickness minimizing the total heating cost is calculated with the equation below.

$$x_{opt,H} = 293.94 \times \left( \frac{HDD \times C_f \times PWF \times k}{H_u \times C_i \times \eta} \right)^{1/2} - k \times R_{wt}$$  \hspace{1cm} (24)

The optimum insulation thickness minimizing the total cooling cost is calculated with the equation below.

$$x_{opt,C} = 293.94 \times \left( \frac{CDD \times C_e \times PWF \times k}{C_i \times COP} \right)^{1/2} - k \times R_{wt}$$  \hspace{1cm} (25)

The optimum insulation thickness minimizing the total heating and cooling cost is calculated with the equation below.
From equation 26, it can be seen that optimum insulation thickness depends on degree-days, cost of electricity and fuel, cost of insulation material, PWF value, and, thermal conductivity for wall and insulation material.

5. ENERGY COST SAVING ANALYSIS

The potential annual energy cost saving per unit area of building exterior walls can be derived by subtracting the annual total energy cost per unit area of building exterior walls for noninsulated walls and the annual total cost per unit area of building exterior walls for insulated walls. The relationship is shown as the following Equation.

\[ E_{CS,H} = C_H - C_{t,H} \]
\[ E_{CS,C} = C_C - C_{t,C} \]

where \( E_{CS,H} \) and \( E_{CS,C} \) is the annual heating energy cost saving per unit area of building exterior walls, ($/m^2); \( C_H \) and \( C_C \) is the annual heating and cooling total cost per unit area of building exterior walls for non-insulated walls, ($/m^2); and \( C_{t,H} \) \( C_{t,C} \) ins is the annual heating and cooling total cost per unit area of building exterior walls for insulated walls, ($/m^2).

Energy savings can be expressed as % by the following equation:

\[ \frac{E_{CS,H,C}}{C_{H,C}} \times 100 \] (29)

6. Payback period

The payback period (years) can be calculated by dividing the total insulation cost, by the yearly energy cost savings, which is given as follows

\[ PP_H = \frac{C_{inc}}{E_{CS,H}} \] (30)
\[ PP_C = \frac{C_{inc}}{E_{CS,C}} \] (31)
Payback period indicates the number of years necessary to recover the investment.

7. CONCLUSION

Heating or cooling degree days is one of the most common methods used to calculated heating and cooling load which is the one of an input parameter used in the determination of optimum insulation thickness of the external wall, energy cost savings over a lifetime and payback periods. The method depends on several parameters such as the base temperature $T_b$ and, the mean daily temperature $T_o$. There is an inverse relationship between the HDD / CDD and the value of the base temperature $T_b$. So the $T_b$ increases the corresponding value of CDD / HDD decreases.

Also heating or cooling degree day’s technique applied to calculate heating and cooling load, with the most commonly economical analysis methods used to optimize the thermal insulation thickness of external walls is the Life Cycle Cost (LCC) analysis.

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