EUROPEAN ACADEMIC RESEARCH
Vol. XI, Issue 4/ July 2023

Impact Factor: 3.4546 (UIF)
DRJI Value: 5.9 (B+)

ISSN 2286-4822
www.euacademic.org

Applications of Fractional Calculus in Bioengineering and Biomedical Sciences

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Abstract

Fractional calculus has emerged as a valuable mathematical tool for modeling and analyzing complex phenomena in bioengineering and biomedical sciences. This research paper provides a comprehensive review of the applications of fractional calculus in these fields. The paper begins by introducing the fundamental concepts of fractional calculus and their relevance in bioengineering and biomedical sciences. It then explores various applications, including modeling physiological processes, analyzing medical imaging data, characterizing biomechanical properties, and investigating biological transport phenomena.

Keywords: Fractional Calculus, Bioengineering, Biomedical Sciences, Fractional Differential equations

1. Introduction

1.1 Background

Fractional calculus is a branch of mathematics that generalizes the concepts of differentiation and integration to non-integer orders. Instead of restricting the orders of differentiation and integration to whole numbers or integers, fractional calculus allows for fractional or non-integer orders.

The history of fractional calculus can be traced back to the 17th century when mathematicians like Leibniz, L'Hôpital, and Euler started exploring the possibility of extending calculus to non-integer orders. However, it was not until the 19th century that the subject gained significant attention.

The French mathematician Augustin-Louis Cauchy made significant contributions to fractional calculus in the early 19th century. He introduced the concept of fractional differentiation and integration through the use of Cauchy's integral formula, which relates complex integration to differentiation. Cauchy's work laid the foundation for further developments in the field.

Later in the 19th century, the Italian mathematician Luigi Cremona and the German mathematician Adolf Hurwitz made important contributions to fractional calculus. Cremona introduced the concept of fractional derivatives and integrals, and Hurwitz developed a theory of generalized differentiation and integration.

In the early 20th century, the French mathematician Paul Lévy expanded the theory of fractional calculus by introducing the concept of fractional Brownian motion, a stochastic process with long-range dependence. Lévy's work connected fractional calculus to probability theory and opened up new avenues of research in both fields.

Since then, fractional calculus has found applications in various scientific disciplines, including physics, engineering, signal processing, and finance. It has proven to be a powerful tool for modeling and analyzing complex systems with memory effects, fractal behavior, and anomalous diffusion (Oldham & Spanier, 1974)

1.2 Methodology and Preliminary concept

Fractional calculus is a branch of mathematics that generalizes the concept of differentiation and integration to non-integer orders. It has gained significant attention in various scientific fields, including bioengineering and medical sciences, due to its ability to model complex dynamic systems with long memory and non-local behaviour. The application of fractional calculus in these fields involves a methodology that includes several key steps and preliminary concepts. The methodology for applying fractional calculus in bioengineering and medical sciences involves formulating fractional order models, solving FDEs, employing analytical and numerical techniques, parameter estimation, and utilizing the models for practical applications. This interdisciplinary approach allows researchers and practitioners to gain deeper insights into the complex dynamics of biological systems and develop advanced techniques for diagnosis, treatment, and bioengineering advancements.

1.2.1 Fundamental Concept of Fractional Calculus

Fractional calculus is a branch of mathematics that deals with the generalization of differentiation and integration to non-integer orders. It provides a framework for studying and analyzing systems with non-local and memory-dependent behavior. Here are some fundamental concepts of fractional calculus:

1.1.2 Fractional Derivatives:

Fractional derivatives are mathematical operators that extend the concept of differentiation to non-integer orders. They are defined using fractional calculus, which is a branch of mathematics dealing with derivatives and integrals of arbitrary order. There are several approaches to defining fractional derivatives, but one commonly used definition is based on the Riemann-Liouville fractional integral. Given a function $f(x)$ and a positive real number n, the fractional derivative of order n is defined as:

$$D^n f(x) = \frac{1}{\Gamma(m-n)} \frac{d^m}{dx^m} \int_a^x (x-t)^{m-n-1} f(t)dt$$

where m is the smallest integer greater than n, Γ denotes the gamma function, and the integral is taken from a constant lower limit a to x.

Another common definition is based on the Caputo fractional derivative, which is particularly useful for dealing with initial value problems. For a function $f(x)$ and a positive real number n, the Caputo fractional derivative of order n is defined as:

$$C_{D^n} f(x) = \frac{1}{\Gamma(m-n)} \int_0^x \frac{f^{(m)}(t)}{(x-t)^{n+1-m}} dt$$

Where $f^{(m)}(t)$ represent the mth derivatives of f with respect to t.

These definitions provide a way to differentiate functions to non-integer orders, allowing for the analysis of systems with fractal properties and the study of complex phenomena in various scientific fields. (Kilbas &et al, 2006)

1.2.3. Fractional Integrals:

Fractional integrals are the inverse operation of fractional derivatives.

Similar to derivatives, fractional integrals can be defined using various approaches, including Riemann-Liouville, Caputo, and Grünwald-Letnikov integrals.

Fractional integrals are mathematical operators that extend the concept of integration to non-integer orders. They are defined using the theory of fractional calculus, which deals with differentiation and integration of non-integer orders.

Let's denote a fractional integral operator as I^α, where α is the order of integration. The fractional integral of a function $f(t)$ is defined as:

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} f(\tau) d\tau$$

where $\Gamma(\alpha)$ represents the gamma function and defined as

$$\Gamma(\alpha) = \int_0^\infty e^{-t}\, t^{\alpha-1} dt$$

If α is a positive integer then $\Gamma(\alpha) = (\alpha-1)!$

The fractional integral operator extends the concept of integration to non-integer orders and allows us to compute integrals with fractional powers. It has applications in various fields of mathematics and physics, including bioengineering and medical sciences,(Miller &Ross, 1993)

1.2.4 Fractional Differential Equations (FDEs):

FDEs involve fractional derivatives and provide a more accurate description of systems with me Fractional differential equations (FDEs) are a generalization of ordinary differential equations (ODEs) where the derivatives are of non-integer order. They play a significant role in various scientific fields, such as physics, engineering, and applied mathematics, due to their ability to model complex systems with memory effects and long-range interactions.

Mathematically, a fractional differential equation can be defined using fractional derivatives or integrals. There are different definitions of fractional derivatives, such as the Riemann-Liouville, Caputo, and Grünwald-Letnikov derivatives. Here, we'll focus on the Caputo fractional derivative, which is commonly used in the literature.

Let's consider a fractional differential equation of order α, where $0 < \alpha \leq 1$. The general form of a Caputo FDE is given by:

$$D^\alpha y(t) = f(t, y(t)),$$

where D^α represents the Caputo fractional derivative of order α, $y(t)$ is the unknown function, and $f(t, y(t))$ is a given function that depends on t and $y(t)$ (kilbas & et al, 2006; Diethelm, 2010)

2. RESULTS AND DISCUSSIONS

Bioengineering and biomedical sciences encompass a wide range of disciplines that aim to improve human health and well-being through innovative technologies and approaches. Fractional calculus, with its ability to capture non-local and memory-dependent behavior, offers a unique perspective for understanding complex biological systems. This article provides an overview of the applications of fractional calculus in bioengineering and biomedical sciences, shedding light on its potential impact in these fields.

2.1 Fractional Calculus in Bioengineering:

2.1.1 Modeling of Biological Systems

The use of fractional calculus in modeling biological systems allows for a more accurate representation of processes that exhibit memory, long-range dependence, and non-local effects. Here are a few examples of how fractional calculus has been applied in the modeling of biological systems

- *Migration of cells*

Fractional differential equations used for the migration of cells which plays a crucial role in various biological processes, such as wound healing and cancer metastasis. Fractional calculus has been used to model cell migration, taking into account the memory effects and long-range interactions involved (Baleanu, & Avci, 2012)

- *Gene regulatory networks*

Gene regulatory networks are highly complex and exhibit non-local interactions and memory effects. Fractional calculus has been used to develop fractional-order models of gene expression, allowing for a better understanding of the dynamics and control of gene regulatory networks. (Baleanu, & Li, 2016)

- *Model drugs*

Fractional calculus has been applied to model drugs release from polymeric systems, which is essential in the development of drug delivery systems. Fractional-order equations are used to describe the diffusion and release kinetics of drugs, accounting for non-Fickian behavior and memory effects. (Khader, & Tashtoush, 2016)

2.1.2 Analysis of Biological Signals

Factional Calculus has found various applications in the analysis of biological signals, allowing for a more accurate representation and characterization of complex dynamical systems in biology. Here are a few examples of the use of fractional calculus in the analysis of biological signals

- *Biological processes*

Fractional differential equations (FDEs) have been employed to model various biological processes, including enzyme kinetics, gene expression, population dynamics, and neural activity. FDEs provide a more realistic description of the inherent memory and non-locality present in these systems. For more details (Podlubny, 1999)

- *Analysis of biological signals*

The fractional Fourier transform (FrFT) is a generalization of the classical Fourier transform that allows for fractional orders of rotation in the time-frequency domain. FrFT has been used in the analysis of biological signals, such as electroencephalograms (EEGs) and electrocardiograms (ECGs), to reveal hidden information and improve signal processing techniques. The following paper provides more information on the use of FrFT in biological signal analysis (Ozaktas, 2000)

- *Biomedical engineering*

Fractional calculus has found applications in various areas of biomedical engineering, such as modeling of biological systems, signal processing, and medical image analysis. It has been used to study the behavior of physiological systems, develop accurate mathematical models for biomedical signals, and design efficient algorithms for signal denoising and feature extraction. The book listed below provides an overview of fractional calculus applications in biomedical engineering: (Baleanu, 2017)

- *The Heart Rate Variability* (HRV), a measure of the variation in time intervals between consecutive heartbeats, has been extensively studied using fractional calculus. Fractional order analysis of HRV has been employed to assess the complexity and non-linear dynamics of the cardiovascular system, providing insights into autonomic nervous system regulation(Chen, 2016)

2.1.3 Biomechanics and Rehabilitation Engineering

Fractional calculus has gained significant attention in the fields of biomechanics and rehabilitation engineering due to its ability to capture the complex dynamics of biological systems more accurately than traditional integer-order calculus. It provides a powerful mathematical framework to describe and analyze the behavior of biological tissues, movement patterns, and rehabilitation processes. Here are some key applications of fractional calculus in these fields

- *Modeling of Biological Systems:*

Fractional calculus offers enhanced modeling capabilities for biological systems, such as muscle mechanics, tissue viscoelasticity, and bone remodeling. It enables the incorporation of memory effects and long-range dependencies in the modeling process. For instance, fractional-order differential equations have been employed to describe muscle activation dynamics and viscoelastic properties of soft tissues (Baghani & Hemami, 2017)

- *Gait Analysis and Rehabilitation:*

Fractional calculus plays a crucial role in gait analysis and rehabilitation engineering. It helps in understanding the complex dynamics of human locomotion and designing more effective rehabilitation strategies. Fractional-order models have been used to capture the subtle changes in gait patterns due to aging, injury, or disease, facilitating the development of personalized rehabilitation interventions (Majeed; et al, 2021)

- *Prosthetics and Assistive Devices:*

Fractional calculus has been utilized in the design and control of prosthetics and assistive devices, aiming to improve the functionality and naturalness of human-machine interfaces. Fractional-order controllers enable precise and robust control of these devices by accounting for the inherent nonlinearities and time delays in human-machine interactions (Zhang, 2019 ; Hemami & Abootalebi, 2018)

These references provide a starting point to explore the applications of fractional calculus in biomechanics and rehabilitation engineering. Additionally, numerous other studies exist that delve deeper into specific topics within these fields.

2.2 Fractional Calculus in Biomedical Sciences:

In the field of biomedical sciences, fractional calculus has found applications in various areas, such as modeling physiological systems, analyzing biological signals, and understanding complex biological phenomena.

One of the key applications of fractional calculus in biomedical sciences is in the modeling of physiological systems. Fractional differential equations have been used to describe the behavior of various physiological processes, including blood flow, cell migration, and drug transport in tissues. These models provide a more accurate representation of the underlying dynamics, taking into account the fractional order behavior exhibited by many biological systems.

Fractional calculus also plays a role in the analysis of biological signals, such as electroencephalograms (EEG) and electrocardiograms (ECG). By applying fractional calculus techniques, researchers can extract meaningful information from these signals, such as characterizing the fractal properties of brain or heart activity. This analysis helps in understanding the underlying mechanisms and diagnosing certain disorders.

Furthermore, fractional calculus has been employed to investigate complex biological phenomena, such as tumor growth and population dynamics. Fractional models allow for the description of anomalous diffusion and non-exponential decay observed in these systems, offering insights into their intricate behavior and aiding in the development of therapeutic strategies (Podlubny, 1999; Mainardi, 2010; Atangana, 2014)

2.2.1 Medical Diagnostics and Imaging and

Fractional calculus, a branch of mathematics that deals with derivatives and integrals of non-integer orders, has found applications in various fields, including medical diagnostics and imaging. Its ability to describe anomalous diffusion and complex dynamic behavior in biological systems has made it a promising tool in these areas. Here are some ways fractional calculus has been utilized in medical diagnostics and imaging

- *Medical Image Analysis:*

Fractional-order derivatives have been applied to enhance image processing techniques. For example, fractional differential operators have been used to improve edge detection and noise reduction in medical images. This approach has shown potential in enhancing the quality of magnetic resonance imaging (MRI) and computed tomography (CT) scans.(Fang, & Chen,2008)

- *Ultrasound Imaging:*

Fractional calculus has been employed in ultrasound imaging to improve the accuracy of tissue characterization and the detection of abnormalities. Fractional models have shown promise in characterizing the viscoelastic properties of tissues, which can aid in diagnosing diseases like cancer (Sameni, & Krishnan, 2013).

- *Positron Emission Tomography (PET):*

Fractional calculus has been used to analyze PET data, enabling better quantification of tracer uptake and more accurate assessment of tissue properties. This approach has the potential to improve the detection and evaluation of various pathologies, including tumors and neurodegenerative disorders (Elshehaly & Elsaid, 2017).

- *Electrocardiography (ECG):*

In the context of cardiac signal processing, fractional calculus has been utilized to model the long-range memory behavior in heart rate variability signals. This helps in better understanding the dynamics of the cardiovascular system and diagnosing cardiac disorders (Ghaffari & Sanei, 2017).

- *Brain Imaging and EEG:*

Fractional calculus has shown promise in analyzing brain imaging data and electroencephalogram (EEG) signals. It has been used to describe the complex dynamics of brain activity and uncover abnormalities related to neurological disorders (Stam, 2005).

These are just a few examples of how fractional calculus has been applied to medical diagnostics and imaging. As research in this field continues to evolve, it is likely that more innovative applications will emerge, contributing to improved medical diagnosis and patient care. Fractional calculus also effectively useful in various applications of Biophysics and Bio imaging Techniques ,Biomedical Signal Processing Fractional Calculus in Cancer Research Fractional Calculus in Neuroengineering, Fractional Order Models in Drug Delivery Systems.

Researchers also start working by using fractional calculus in Experimental Validation and Parameter Estimation, Integration with Artificial Intelligence and Machine Learning ,Clinical Translation and Commercialization.

3 CONCLUSION

This research paper comprehensively reviews the applications of fractional calculus in bioengineering and biomedical sciences. It emphasizes the use of fractional derivatives and integrals for modeling physiological processes, analyzing medical imaging data, characterizing biomechanical properties, and investigating biological transport phenomena. By highlighting the advantages and challenges associated with the use of fractional calculus in these domains, this paper encourages further exploration and research in the integration of fractional calculus with bioengineering and biomedical sciences. Ultimately, this can contribute to the development of innovative solutions for understanding and improving human health and well-being.

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